MATH 1113 **Name:**_____

Final Exam

Date: _____ **Instructor:** _____

1. Let $f(x)=\frac{\sqrt{1-x}}{x+9}$.

a. State the domain of f.

b. Find $f(-3)$.

2. Given $f(x)=x^2$, find the average rate of change of f from $x_1 = 3$ to $x_2 = 3+h$. Express your answer in simplest form.

3. The polynomial function $P(x) = x^3 - 6x^2 + 11x - 6$ has only real zeros.

a. Use the factor theorem to determine if 2 a zero of this function.

b. Find the end-behavior of P(x), and sketch its graph.

c. Express *P(x)* as a product of linear factors.

4. The Intermediate Value Theorem for Polynomial Functions: If f is a polynomial function and $[a, b]$ is a closed interval from the domain of f, then f takes on every value between $f(a)$ and $f(b)$ in the interval $[a, b]$.

a. Use the intermediate value theorem to show that $f(x) = x^3 - 2$ has a zero between $x = 1$ and $x = 2$.

b. Use your calculator to estimate the zero to at least two decimal places of accuracy.

5. Given the rational function $r(x) = \dfrac{x^2 + 3x - 4}{x^2 - 9}$

a. State the domain of *r*(x).

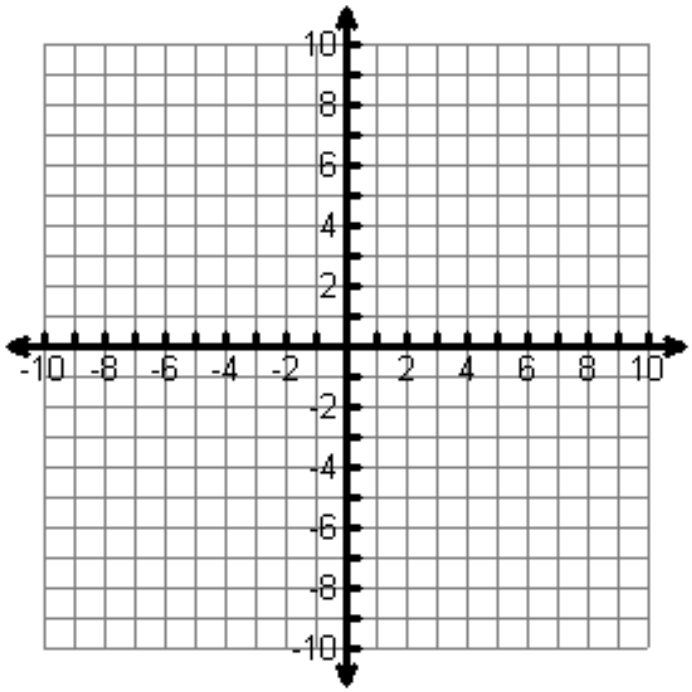
b. Find the vertical asymptotes of the graph of *r*(x).

c. Find the x -intercepts of the graph of *r*(x).

d. Find the y -intercepts of the graph of *r*(x).

e. Find the horizontal asymptote of the graph of *r*(x).

f. Sketch the graph of r(x).



6. Solve the equation. Leave your answer in exact form.

$$3^{x+1} = 2^{2x}$$

7. Solve the equation. Leave your answer in exact form.

$$\ln(x) - \ln(x-1) = 1$$

8. The amount of Carbon 14 (in grams) in a piece of charcoal from a tree killed during an ancient volcanic eruption has been shown by experiment to be

$$Q(t) = 3e^{\frac{-\ln 2}{5730}t}$$

a. Estimate $Q(t)$ if t = 9000 years. (Round your answer to the nearest hundredth of a gram)

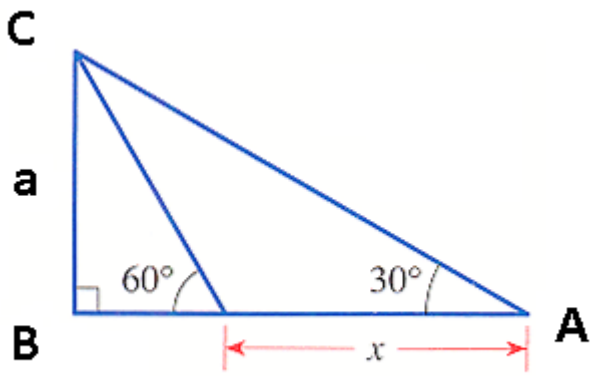
b. Estimate the time t for which $Q(t) = 2$ grams.

9. Let $f(x) = 1 + \frac{3}{x}$ and $g(x) = x^2 + 1$.

a. Find the inverse function of f.

b. Find $(f \circ g)(x)$.

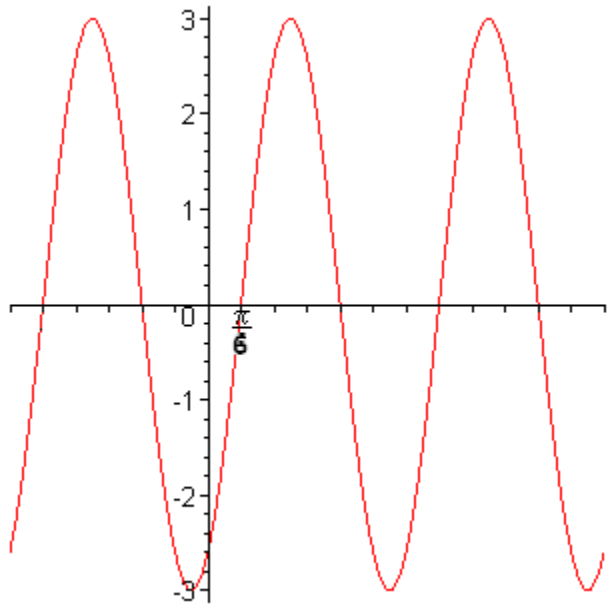
10. If **a**= 20 in the following figure, find the exact value of *x*.



11. From a point 5 meters above level ground, an observer measures the angle of depression to an object on the ground is 69°. Approximate the distance from the object to the point on the ground directly beneath the observer. Round your answer to the nearest tenth of a meter.

12. Find the sign of the expression cot(*t*)tan(*t*) if the terminal point determined by *t* is in the quadrant II.

13. In the following graph of a sine curve, the vertical scale is as shown and the horizontal scale is $\frac{\pi}{6}$.



a.) Determine the amplitude of the curve.

b.) Determine the period of the curve.

c.) Determine the phase shift of the curve.

d.) Determine the function in the form $f(x) = a\sin(k(x-b))$ or $f(x) = a\sin(bx+c)$.

14. Sketch 2 periods of the graph of $y = \tan\left(2x + \frac{\pi}{2}\right)$. Label the asymptotes and the x-intercepts.

15. Verify the following trigonometric identity.

$$\frac{\sec x - \cos x}{\tan x} = \sin x$$

16. Find all solutions of the following equation in the interval $[0, 2\pi)$. Leave your answers in exact form.

$$2\cos^2 x + \cos x = 1$$

17. Find the solutions of the following equation that are in the interval $[0, 2\pi)$. Assume θ is measured in radians and round any answers correct to 2 decimal places.

$$5\cos\theta - 3 = 0$$

18. Find the exact value of the following expressions.

a. $\tan\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$

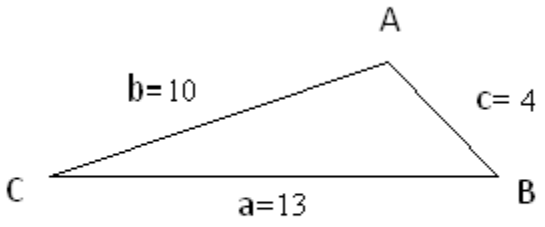
b. $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

19. Given $\sec(\theta) = -\frac{5}{4}$ *and* $\pi \le \theta \le \frac{3\pi}{2}$, find exact values for the following expressions.

a. $\sin 2\theta$

b. $\cos\frac{\theta}{2}$

20. Use the law of sines or the law of cosines to find the angles in the following triangle. State the angles in degrees to the nearest hundredth of a degree.



Addition/Subtraction Identities

$\sin(u+v) = \sin u \cos v + \cos u \sin v$ $\quad\quad$ $\cos(u+v) = \cos u \cos v - \sin u \sin v$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$ $\quad\quad$ $\cos(u-v) = \cos u \cos v + \sin u \sin v$

Double Angle Identities

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u$

$\cos 2u = 2\cos^2 u - 1$

$\cos 2u = 1 - 2 \sin^2 u$

Half Angle Identities

$\sin \frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$ $\quad\quad$ $\cos \frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, where *a, b, c* are lengths of sides and *A, B, C* are the opposite angles.

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$, where *a, b, c* are lengths of sides and *A* is the angle opposite side *a*.