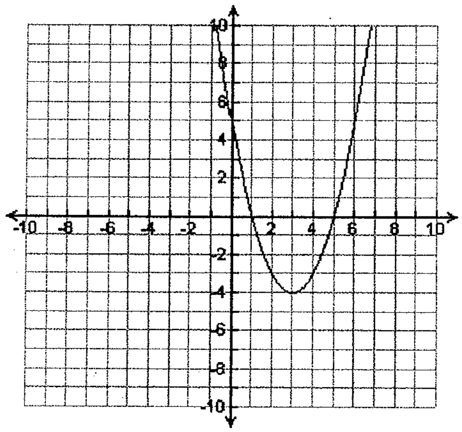
MATH 1113 **Final Exam** **Name:** Solutions
Date: May 3, 2011 **Instructor:** ____________

1. Given the following graph of a function f,



(4 pt.) **a.** Find the domain of f.

dom(f) = $(-\infty, \infty)$

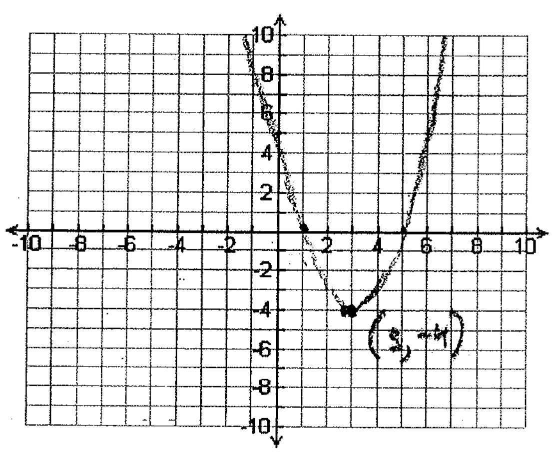
(4pt.) **b.** Find the range of f.

ran(f) = $[-4, \infty) = \{y \mid y \geq -4\}$

(2 pt.) **c.** Find f(6).

2. Given $f(x) = x^2$, let $g(x) = f(x-3) - 4$.

Sketch the graph of g(x) below, labeling the vertex and x-intercepts of the graph.
(Graph 8 pt., vertex 2 pt.)



(10 pt.) **3.** Given $f(x) = x^2 + x$, find the average rate of change of f from x = 2 to x = 3.

$$\frac{f(3) - f(2)}{3 - 2} = \frac{[(3^2+3) - (2^2+2)]}{1} = 12 - 6 = \boxed{6}$$

(10 pt.) **4.** Given the polynomial function $P(x) = x^3 - 2x^2 - 5x + 6$, use the factor theorem to determine whether $x-1$ is a factor of $P(x)$. If so, express $P(x)$ as a product of linear factors.

$$P(1) = 1^3 - 2(1)^2 - 5(1) + 6$$
$$= 7 - 7 = 0$$

Yes, $x-1$ is a factor of $P(x)$.

Factorization:

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$(x-1)(x^2 - x - 6) = \boxed{(x-1)(x+2)(x-3)}$$

5. The Intermediate Value Theorem for Polynomial Functions: If f is a polynomial function and $[a,b]$ is a closed interval from the domain of f, then f takes on every value between $f(a)$ and $f(b)$ in the interval $[a,b]$.

(6 pt.) **a.** Use the intermediate value theorem to show that $f(x) = x^5 - 2$ has a zero between $x=1$ and $x=2$.

Since $f(1) = 1^5 - 2 = -1$

and $f(2) = 2^5 - 2 = 30$

and $-1 < 0 < 30$, then there must be a zero between $x = 1$ and $x = 2$.

(4 pt.) **b.** Use your calculator to estimate the zero to at least two decimal places of accuracy.

$$\boxed{x \approx 1.15}$$

6. Given the rational function $r(x) = \dfrac{x+1}{x-2}$

(2 pt.) **a.** Find the equation of the vertical asymptote of the graph of $r(x)$.

$\boxed{x = 2}$

(2 pt.) **b.** Find the x and y intercept(s) of the graph of $r(x)$.

$r(0) = \dfrac{0+1}{0-2} = -\dfrac{1}{2}$

$\dfrac{x+1}{x-2} = 0 \rightarrow x+1 = 0 \rightarrow x = -1$

$\boxed{\text{y-intercept: } (0, -\tfrac{1}{2})}$

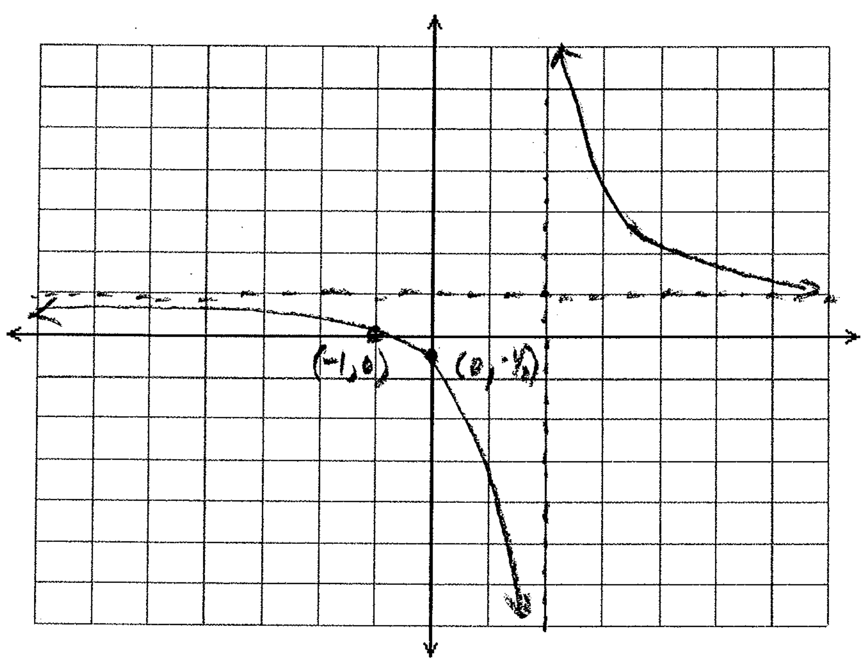
$\boxed{\text{x-intercept: } (-1, 0)}$

(2 pt.) **c.** Find the equation of the horizontal asymptote of the graph of $r(x)$.

$r(x) = \dfrac{x+1}{x-2} = \dfrac{1 + 1/x}{1 - 2/x} \rightarrow 1$ as $x \rightarrow \infty$

horizontal asymptote: $\boxed{y = 1}$

(4 pt.) **d.** Sketch the graph of $r(x)$.



7. Given $f(x)=\cos(x)$ and $g(x)=x^2$, find the following.

(6 pt.) **a.** $(f \circ g)(x)$ $= f(g(x)) = \boxed{\cos(x^2)}$

(4 pt.)**b.** $(g \circ f)(\pi)$ $= g(f(\pi)) = (\cos\pi)^2 = (-1)^2 = \boxed{1}$

(10 pt.) **8.** Solve the equation. Leave your answer in exact form.

$\log(x)-\log(x-48)=1$

$\log\left(\frac{x}{x-48}\right) = 1 \iff 10^1 = \frac{x}{x-48}$

$10(x-48) = x$

$10x - 480 = x$

$-480 = -9x$

$x = \frac{480}{9} = \frac{160}{3}$

$\boxed{\text{Solution set: } \left\{\frac{160}{3}\right\}}$

9. The number of bacteria on a certain culture increases exponentially over time. The number f(t) of bacteria after t hours is given by

$$f(t)=200e^{0.75t}$$

(2 pt.) **a. What is the initial number of bacteria? (t=0)**

$f(0) = 200e^{0.75 \cdot 0} = \boxed{200}$

(4 pt.) **b.** Estimate the number of bacteria in the culture after two hours. Round your answer to the nearest whole number.

$f(2) = 200e^{0.75 \cdot 2} = \boxed{896}$

(4 pt.) **c. How long will it take for the number of bacteria to become 4000? Round your answer to the nearest whole number of hours.**

$200e^{.75t} = 4000$

$e^{.75t} = \frac{4000}{200} = 20$

$\ln(e^{.75t}) = \ln(20)$

$.75t = \ln(20)$

$t = \frac{\ln(20)}{.75} \approx \boxed{4 \text{ hrs}}$

(10 pt.) **10.** Find the inverse function of $f(x)=\dfrac{x-2}{x+3}$.

$$y = \frac{x-2}{x+3}$$

$$x \leftrightarrow y$$

$$x = \frac{y-2}{y+3}$$

$$x(y+3) = y-2$$

$$xy + 3x = y - 2$$

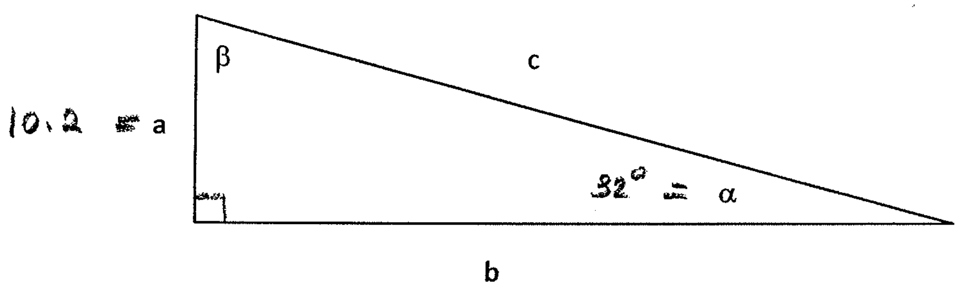
$$3x + 2 = y - xy$$

$$= y(1-x)$$

$$y = \frac{3x+2}{1-x}$$

$$\boxed{f^{-1}(x) = \frac{3x+2}{1-x}}$$

11. Solve the right triangle for angle β and sides b and c, given that $\alpha = 32^\circ$ and side a = 10.2. Round sides to one decimal place. (angle 2 pt., sides 4 pt. ea.)



$$\sin 32^\circ = \frac{10.2}{c}$$

$$\rightarrow c = \frac{10.2}{\sin 32^\circ}$$

$$\boxed{c \approx 19.2}$$

$$\boxed{\beta = 90^\circ - 32^\circ = 58^\circ}$$

$$\tan 32^\circ = \frac{10.2}{b}$$

$$\rightarrow b = \frac{10.2}{\tan 32^\circ}$$

$$\boxed{b \approx 16.3}$$

12. Find the exact values of the following.

(4 pt.) **a.** $\sec\left(\dfrac{5\pi}{3}\right) = \dfrac{1}{\cos\left(\frac{5\pi}{3}\right)} = \dfrac{1}{\left(\frac{1}{2}\right)} = \boxed{2}$

(4 pt.) **b.** $\cos\left(\dfrac{7\pi}{6}\right) = \boxed{\dfrac{-\sqrt{3}}{2}}$

(2 pt.) **c.** $\tan(3\pi) = \dfrac{\sin(3\pi)}{\cos(3\pi)} = \dfrac{0}{-1} = \boxed{0}$

13. Given the trigonometric function

$$f(x) = 2\sin 2\left(x - \frac{\pi}{2}\right) = 2\sin(2x - \pi)$$

(2 pt.) **a.** Determine the amplitude of *f(x)*.

$\boxed{\text{amplitude} = 2}$

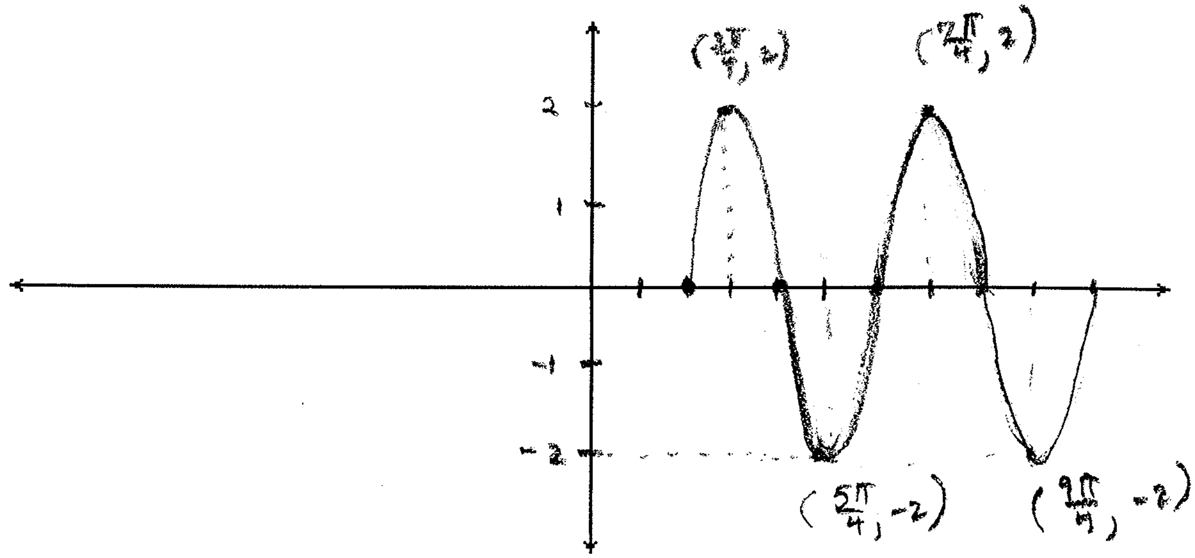
(2 pt.) **b.** Determine the period of *f(x)*.

$\frac{2\pi}{2} = \pi$ $\boxed{\text{period} = \pi}$

(2 pt.) **c.** Determine the phase shift of *f(x)*.

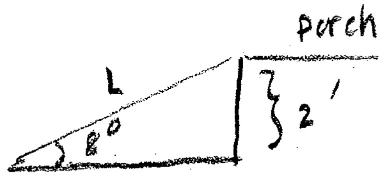
$\boxed{\text{phase shift} = \frac{\pi}{2}}$

(4 pt.) **d.** Sketch two periods of the graph of *f(x)*.



Scale $= \frac{\pi}{4}$

(10 pt.) **14.** A ramp is to be built to access a porch that is 2 feet high. How long must the sloped part of the ramp be if it makes an 8° angle with the ground?



$$\sin 8^\circ = \frac{2}{L}$$

$$L = \frac{2}{\sin 8^\circ} \approx \boxed{14.37 \text{ ft}}$$

(10 pt.) **15.** Verify the following trigonometric identity.

$$\frac{\cos(t)}{1-\sin(t)} - \frac{\sin(t)}{\cos(t)} = \sec(t)$$

$$\frac{\cos t}{1-\sin t} \cdot \frac{\cos t}{\cos t} - \frac{\sin t}{\cos t} \cdot \frac{1-\sin t}{1-\sin t}$$

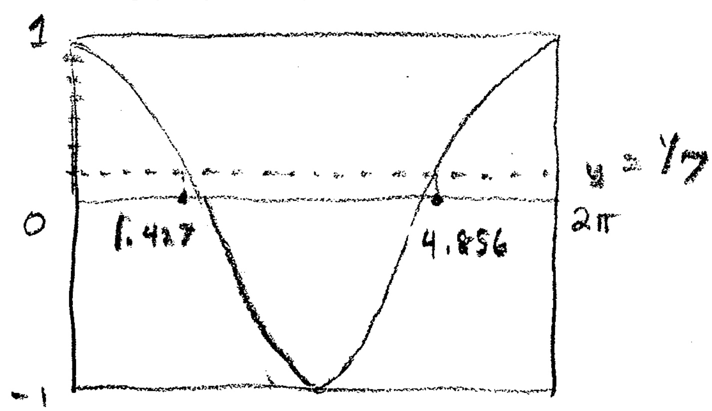
$$= \frac{\cos^2 t - \sin t + \sin^2 t}{\cos t\,(1-\sin t)}$$

$$= \frac{\cancel{1-\sin t}}{\cos t\,(\cancel{1-\sin t})} = \frac{1}{\cos t} = \sec t \; ✓$$

(10pt.) **16.** Solve the following equation in the interval $[0, 2\pi)$. State your answers correct to three decimal places.

$$7\cos(x) - 1 = 0$$

$$\cos x = \frac{1}{7}$$



$$\boxed{x = 1.427,\ 4.856}$$

(10 pt.) **17.** Find all solutions to the following equation. Give your answers in exact form.

$\cot(\theta) = 1$

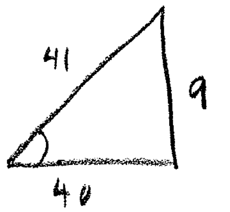
$$\boxed{\theta = \frac{\pi}{4} + n\pi, \; n = 0, \pm 1, \pm 2, \ldots}$$

$$\frac{\cos\theta}{\sin\theta} = 1$$

$$\cos\theta = \sin\theta$$

18. Find the exact value of the following expressions.

(6 pt.) **a.** $\cos\left(\tan^{-1}\left(\frac{9}{40}\right)\right) = \boxed{\frac{40}{41}}$

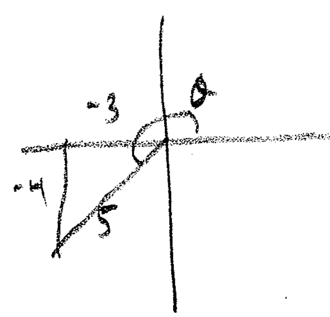


(4 pt.) **b.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$

19. If $\sin(\theta) = -\frac{4}{5}$, and θ is an angle in quadrant III, find the exact value of the following.

(6 pt.) **a.** $\sin\left(\frac{\theta}{2}\right)$

$$\pi < \theta < \frac{3\pi}{2} \longrightarrow \frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4} \text{ (quadrant II)}$$



$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1}{2}(1-\cos\theta)} = \sqrt{\frac{1}{2}\left(1+\frac{3}{5}\right)}$$

$$= \sqrt{\frac{4}{5}} = \boxed{\frac{2\sqrt{5}}{5}}$$

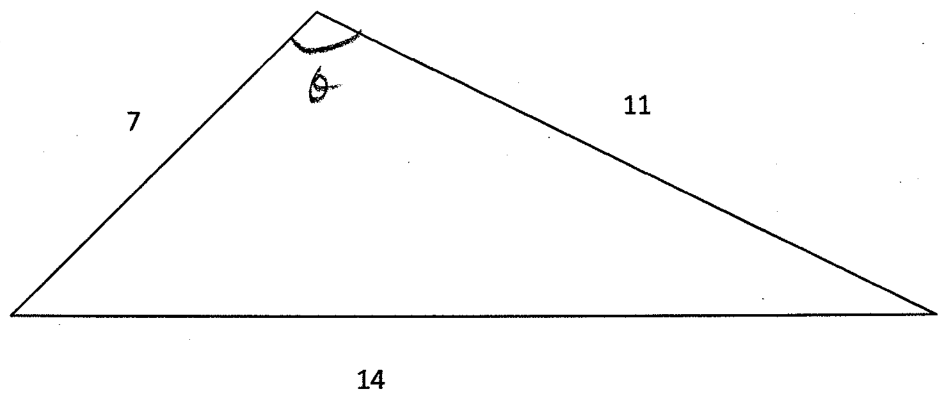
(4 pt.) **b.** $\cos(2\theta)$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$= 2\left(-\frac{3}{5}\right)^2 - 1$$

$$= 2 \cdot \frac{9}{25} - 1 = \frac{18}{25} - \frac{25}{25} = \boxed{-\frac{7}{25}}$$

20. Refer to the figure below, depicting a triangle with sides shown.



(6 pt.) **a.** Find the angle (in degrees) opposite the longest side. State your answer to the nearest tenth.

Law of cosines

$$14^2 = 7^2 + 11^2 - 2(7)(11)\cos\theta$$

$$\cos\theta = \frac{26}{-156}$$

$$\theta = \cos^{-1}\left(-\frac{26}{156}\right) \approx \boxed{99.6^\circ}$$

(4 pt.) **b.** Find the area of the triangle. State your answer to the nearest tenth.

$$A = \frac{1}{2}(7)(11)\sin(99.6^\circ)$$

$$\approx \boxed{38}$$

or using Heron's formula

$$S = \frac{7+11+14}{2} = 16$$

$$A = \sqrt{16(16-7)(16-11)(16-14)}$$

$$\approx 37.94 \approx \boxed{38}$$