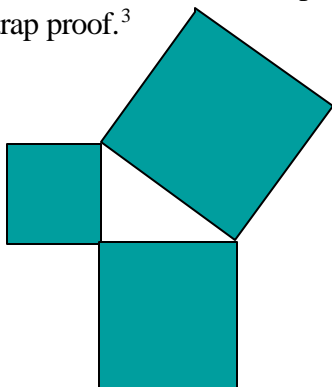


The Scarecrow Conjecture Activity

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In the introduction to the original text, The Wonderful Wizard of Oz Frank Baum claims that his intent was to write "a modernized fairy tale", however, it is clearly an allegory. Though somewhat altered, the allegorical nature of the story is preserved in the popular motion picture adaptation, The Wizard of Oz. In the movie, when the wizard gives the scarecrow a diploma, the scarecrow proudly rattles off something that sounds like the Pythagorean Theorem, but it is not. The actual Pythagorean Theorem states that the square of the length of the hypotenuse in a right triangle is equal to the sum of the squares of the lengths of the other two sides. The scarecrow states, "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.", a statement which we have named "the scarecrow conjecture".

Until a statement has been proved, it remains as a conjecture or hypothesis. Once it has been proved, it is a theorem. For example, the Pythagorean Theorem is called a theorem because it has been proved. This relationship between sides of right triangles was known to be true even before the time of Pythagorus (580-500 BCE) as established by the Babylonian artifact known as Plimpton 322 (1900-1600 BCE)⁴ which contains known Pythagorean triples (integers related to each other by the Pythagorean relationship). The theorem was later named in honor of Pythagorus but many proofs were discovered independently by different people in different cultures. One such proof can be found as Proposition 47 in Book I of Euclid's Elements.⁶ This particular geometric proof is known as the bride's chair proof or the mousetrap proof.³



The symbol a^2 is called "a-squared" because of its relationship to the area of a square of side length "a". If you slice the diagram above with squares of sides of lengths a , b , and c , with c being the length of the side of the largest (tilted) square, it is clear, though not trivial, that for the arbitrary right triangle seen in the center, $a^2 + b^2 = c^2$. The diagram shows the proof of the relationship.

As an aside, a more generalized form of the Pythagorean Theorem $a^n + b^n = c^n$ has no integer solutions for $n > 2$. This theorem, known as Fermat's Last Theorem, was recently proved by Andrew

J. Wiles.

The scarecrow's claim, that the "sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side", is false. It takes just one counterexample to disprove the general statement. For such a counterexample, take a triangle with all sides of length 1. Since $1 + 1 \dots 1$, the conjecture is false.

After further investigation, we realized that not only is the conjecture not true in general, it can never be true. This led us to the proof of a conjecture which we have named The Crow Theorem.

The Crow Theorem: *The sum of the square roots of any two sides of an isosceles triangle is not equal to the square root of the remaining side.*

Proof by Contradiction. Consider any isosceles triangle with sides of lengths a , a , and b with $2a > b > 0$. Assume that the sum of the square roots of two sides of the triangle equals the square root of the remaining side. We consider two cases:

Case (i): $\sqrt{b} = \sqrt{a} + \sqrt{a}$

$$\sqrt{b} = 2\sqrt{a}$$

$$b = 4a > 2a > b$$

$b > b$ is false.

Case(ii): $\sqrt{a} = \sqrt{a} + \sqrt{b}$

$$0 = \sqrt{b}$$

$0 = b$ is a contradiction. \square

The characters in the book and the movie spend a lot of time searching for qualities which they already possess, the happiness that is actually in their own backyard.² The scarecrow represents the idealistic Kansas farmer of the late 1800's. Though these farmers would have a lot of common sense, they may not feel intelligent because of their lack of formal education. In neither the book nor the movie could the wizard actually give the scarecrow a brain. In each the wizard offers a substitute consolation prize. In the movie this substitute is a diploma, the symbol of a formal education. To demonstrate the impact of this artificial education, the scarecrow tries to state some mathematics, the quintessence of knowledge.

We have a theory as to why the script writers for the movie, chose something close to the Pythagorean Theorem for the scarecrow's mathematics line. We think they wanted to use recognizable mathematics so the viewing audience could better appreciate what the scarecrow was doing. A statement close to the Pythagorean Theorem would be simple to state and it would sound vaguely familiar to many people.

The script writers Noel Langley, Florence Ryerson, Edgar Allan Woolf or perhaps the actor Ray Bolger⁵, either accidentally botched the theorem or deliberately sabotaged it. We feel it was an act of deliberate sabotage because of the speed at which the actor states his lines, suggesting a lot of practice, and the three obvious errors in the wording of the lines. The scarecrow said "square root" instead of "square". Although that was an easy mistake to make, it was not the only mistake. The most suspicious error occurred when he said "isosceles triangle" instead of "right triangle". This seems doubtfully an

accident. Finally, he said, "any two sides" when he needed to designate the hypotenuse. Taking all three errors together with such a common and short theorem, it seems to us deliberately altered.

Did the script writers themselves deliberately alter the theorem or were they simply the victim of a diabolical or incompetent mathematical consultant? If they deliberately altered the theorem, did they knowingly change it to a statement that can never be true? Were they trying to make a point about their view of the real value of diplomas? Were they trying to make a statement about the lack of real knowledge in the population of viewers at large, implying that we are all "scarecrows" as their little inside joke? Was there a "scarecrow conspiracy"? We continue to search for the answer to those questions.

List of References

Frank Lyman Baum, The Wonderful Wizard of Oz, Ballantine, New York, 1900.

Frank Lyman Baum, The Wizard of Oz, (The Motion Picture), Directors: Victor Fleming, King Vidor; Script writers: Noel Langley, Florence Ryerson, Edgar Allan Woolf; Adaptation: Noel Langley; Loew's Inc., 1939

David M. Burton, The History of Mathematics, An Introduction, 3rd ed., McGraw-Hill, 1997, pp. 152-154

Howard Eves, An Introduction to the History of Mathematics, 6th ed., Saunders, New York, 1990.

Sharon K. Hall, Twentieth-Century Literary Criticism, Gale Research, Detroit, vol. 7, pp 11- 19.

David E. Joyce, Euclid's Elements, Book I, Clark University, <http://aleph0.clarku.edu/~djoyce/java/elements/bookI/propI47.html>, 1996

Victor J. Katz, A History of Mathematics, An Introduction, 2nd ed., Addison Wesley, Reading, 1998.