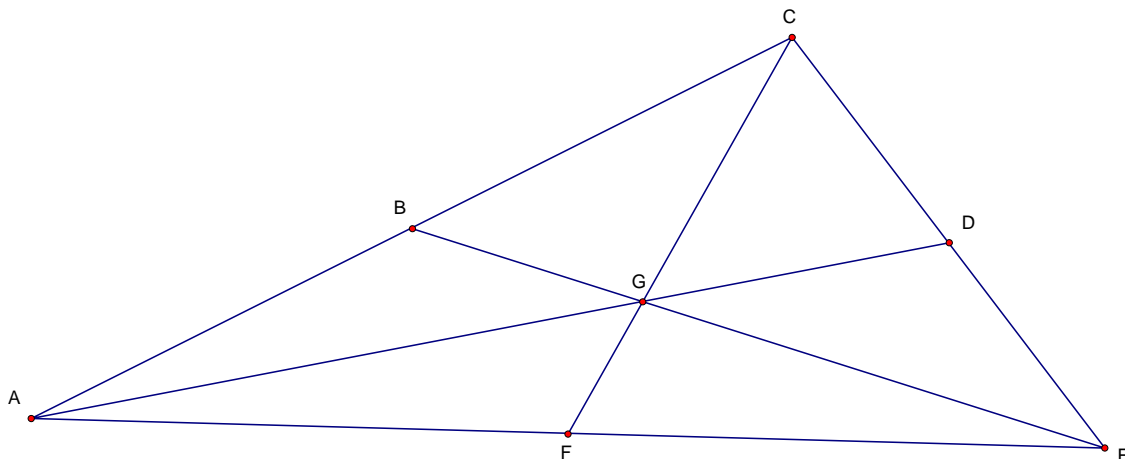


Triangles within a Triangle

When the three medians of triangle are drawn, six non-overlapping triangles are formed. How do the areas of these six triangles compare?



Points F, D, and B are the midpoints of segments AE, EC, and AC, respectively. Thus, segments CF, AD, and EB are medians of the triangle.

When I go to GSP, I find that the areas of the 6 non-overlapping triangles are equal. Click [here](#) for GSP sketch. Note that I am **not** claiming the triangles are congruent—I am merely observing the areas are equal.

Thus, based on my observations I **conjecture** that the three medians of a triangle divide the triangle into 6 non-overlapping triangles that have the same area. I will use the following questions to help you PROVE (using a deductive argument) that these 6 triangles are, in fact, equal in area.

Consider $\triangle AGF$. Using segment AF as the base of this triangle, how would you find the height to this base? How could you find the area of this triangle?

Consider $\triangle EGF$. Using segment FE as its base, how would you find the height to this base? How could you find the area of this triangle?

How does the base of $\triangle AGF$ and the base of $\triangle EGF$ compare? How do the heights drawn to the base of each triangle compare? What conclusion can you make about the areas of $\triangle AGF$ and $\triangle EGF$?

To what other pairs of triangles can you apply similar reasoning? What conclusions can you make? ANSWER THESE QUESTIONS AND JUSTIFY YOUR CONCLUSIONS BEFORE YOU READ ANY FURTHER!!!!

Now that you have made your conclusions, we will continue.....

Let x represent the area of $\triangle AGF$. Based on your work above, what other triangle has an area represented by x ?

Let y represent the area of $\triangle AGB$. What other triangle has an area of y ?

Let z represent the area of $\triangle EGD$. What other triangle has an area of z ?

Consider $\triangle ACF$ and $\triangle ECF$.

How could you represent the areas of $\triangle ACF$ and $\triangle ECF$ in terms of x , y , and z ?

If you consider segments AF and FE as the bases of each triangle, respectively, and then determine the height drawn to each base, what conclusion can you make about their areas? Why?

What conclusion can you make about y and z ? Why?

Represent the areas of $\triangle EAD$ and $\triangle CAD$ in terms of x , y , and z . What do you know about these areas? Why? JUSTIFY!!!!!!

What conclusions can you make about x , y , and z ? Why?

What conclusion can you make about the areas of the 6 non-overlapping triangles determined by the medians of a triangle? Why?