GSP Assignment
Centers of a Triangle Extended

In the activities below, the points G, H, C, and I refer respectively to the centroid, orthocenter, circumcenter, and incenter of a triangle. After examining the activities in the assignment, submit a write-up for problem 5 and at least one other problem.

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1. Take any triangle. Construct a triangle connecting the three midpoints of the sides. This is called the MEDIAL triangle. It is similar to the original triangle and one-fourth its area. Construct G, H, C, and I for this new triangle. Compare to G, H, C, and I in the original triangle.

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2. Take any acute triangle. Construct a triangle connecting the feet of the altitudes. This is called the ORTHIC triangle. Construct G, H, C, and I for the orthic triangle. Compare to G, H, C, and I in the original triangle. Can you extend this to right triangles or obtuse triangles?

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3. Take an acute triangle ABC. Construct the orthocenter, H, and the segments HA, HB, and HC. Construct the midpoints of HA, HB, and HC. Connect the midpoints to form a triangle. Prove that this triangle is similar to triangle ABC and congruent to the medial triangle. Construct G, H, C, and I for this triangle. Compare.

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4. In the same original triangle, construct the three secondary triangles of Exercises 1, 2, and 3. Construct the circumcircle for each of the secondary triangles. What do you observe? Can you prove your conjecture?

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5. The Nine-Point circle for any triangle passes through the three mid-points of the sides, the three feet of the altitudes, and the three mid-points of the segments from the respective vertices to orthocenter. Construct the nine points, locate the center (N) and construct the nine point circle.

How is N related to G, H, C, or I for different shaped triangles?

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7. Prove that the three perpendicular bisectors of the sides of a triangle are concurrent.

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8. Prove that the lines of the three altitudes of a triangle are concurrent.

9. Prove that the three medians of a triangle are concurrent and that the point of concurrency, the centroid, is two-thirds the distance from each vertex to the opposite side.

How would you use GSP to help students understand this relationship of the triangle and its medians? How would you develop a sense of proof of the relationship with students?

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10. Prove that the three angle bisectors of the interior angles of a triangle are concurrent.

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11. Prove that any angle bisector of a triangle is concurrent with the two angle bisectors of the opposite exterior angles of a triangle.

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12. Take a point of concurrency as determined in Problem 11 and construct a circle tangent to the lines of the three sides (of the triangle).

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13. Prove that for any triangle, H, G, and C are collinear, and prove that $HG = 2GC$.