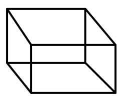
QUAN 6610

Design of Experiments



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DESIGN OF EXPERIMENTS

Purposeful changes of the inputs (factors) to a process in order to observe corresponding changes in the output (response).

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Douglas Montgomery, Design and Analysis of Experiments

Why use DOE ?

- A basis of action -- allows purposeful changes.
- An analytic study -- one in which action will be taken on a cause-and-effect system to improve performance of a product or process in the future.
- Follows the scientific approach to problem solving.
- Provides a way to measure natural variation.
- Permits the clear analysis of complex effects.
- Most efficient way to derive the required information at the least expenditure of resources.

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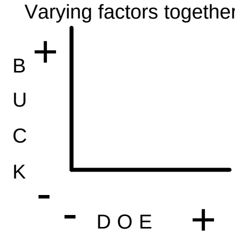
Moen, Nolan and Provost, Improving Quality Through Planned Experimentation

~~Interactions~~

Varying factors together vs. one at a time.



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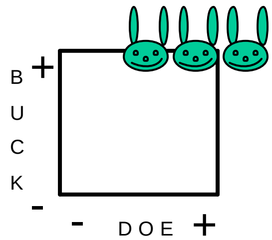
George Box, Do Interactions Really Matter, Quality Engineering,1990.

Voila!



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George Box, Do Interactions Really Matter, Quality Engineering,1990.

Industry Example

- Experiment run at SKF -- largest producer of rolling bearing in the world.
- Looked at three factors: heat treatment, outer ring osculation and cage design.
- Results:
  - choice of cage design did not matter (contrary to previously accepted folklore -- considerable savings)
  - life of bearing increased five fold if osculation and heat treatment are increased together -- saved millions of dollars !

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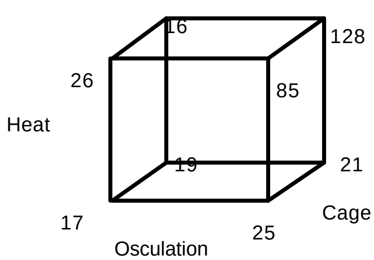
George Box, Do Interactions Really Matter, Quality Engineering,1990.

- Bearings like this have been made for decades. Why did it take so long to discover this improvement ? One factor vs. interaction effects !



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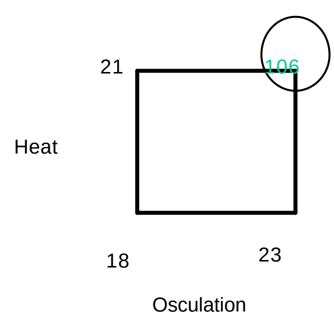
George Box, Do Interactions Really Matter, Quality Engineering,1990.

The Power of Interactions !



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George Box, Do Interactions Really Matter, Quality Engineering,1990.

2^2 Design Example

Consider an investigation into the effect of the concentration of the reactant and the amount of catalyst on the reaction time of a chemical process.

	L	H
reactant (factor A)	15%	25%
catalyst (factor B)	1 bag	2 bags

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Design Matrix for 2^2

A	B	AB	Total	Average
-	-	+		
+	-	-		
-	+	-		
+	+	+		

Main effects

Interaction

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		Replicates			
		I	II	III	Total
Factor	A - B -	28	25	27	80
Settings	A + B -	36	32	32	100
	A - B +	18	19	23	60
	A + B +	31	30	29	90

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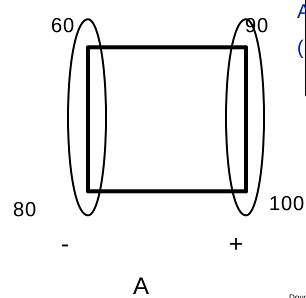
An effect is the difference in the average response at one level of the factor versus the other level of the factor.



A effect =

([90 + 100] - [60 + 80])

/ 2(3) = 8.33

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Use a matrix to find the effects of each factor, including the interaction effect between the two factors.

A	B	AB	Total	Average
-	-	+	80	26.7
+	-	-	100	33.3
-	+	-	60	20
+	+	+	90	30

Avg +	31.7
Avg -	23.3
Effect	8.4

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Completing the matrix with the effect calculations:

A	B	AB	Total	Average
-	-	+	80	26.7
+	-	-	100	33.3
-	+	-	60	20
+	+	+	90	30

	A	B	AB
Avg +	31.7	25	28.3
Avg -	23.3	30	26.7
Effect	8.4	-5	1.7

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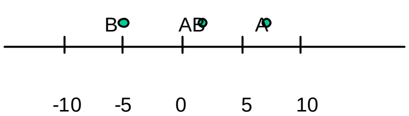
Dot Diagram



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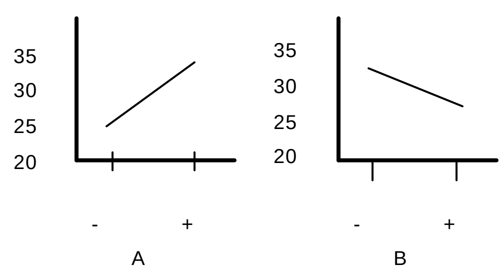
Response Plots



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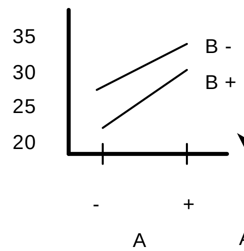

Interaction Response Plot

	B -	B +
A -	26.7	20
A +	33.3	30

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Prediction Equation

The ‘intercept’ in the equation is the overall average of all observations.

The coefficients of the factors in the model are 1/2 the effect.

$Y = 27.5 + 8.33/2\ A - 5/2\ B + 1.7/2\ AB$

or

$Y = 27.5 + 4.165\ A - 2.5\ B + 0.85\ AB$

note: A and B will be values between -1 and +1.

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Analysis of Variance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
A	208.33	1	208.33	53.15 *
B	75.00	1	75.33	19.13 *
AB	8.33	1	8.33	2.13
Error	31.34	8	3.92	
Total	323.00	11		

* = significant at 1% (see F table)

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Calculating SS, df and MS for Effects and Interactions

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
A	208.33	1	208.33	53.15 *

always 1 for this type design

SS = Effect ^{2} x n
= 8.33^2 x 3
where n = replicates

SS / df

Use this same process for A, B and AB

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Calculating total sum of squares and total degrees of freedom

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
Total	323.00	11		

Total df = n - 1 = 12 - 1 = 11

This is found by adding up every squared observation and then subtracting what is called a correction factor (sum of all obser vations square this amount, then divide by the number of observations).

SST = $28^2 + 25^2 + 27^2 + \ldots + 29^2 - (330^2 / 12)$
= 9398.0 - 9075.0 = 323.0

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Calculating error sum of squares, df and mean square

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
Error	31.34	8	3.92	

Found by subtraction:
Total SS - SS_A - SS_B - SS_{AB}
= 323 - 208.33 - 75.0 - 8.33
= 31.34

SS / df

Found by subtraction:
= Total df - A df - B df - AB df
= 11 - 1 - 1 - 1 = 8

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Calculating F ratios

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F
A			208.33	53.15 *
B			75.33	19.13 *
AB			8.33	2.13
Error			3.92	

208.33 / 3.92
75.33 / 3.92
8.33 / 3.92

F ratios:

F = MS (A or B or AB)

MS (error)

Compare to F table

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Interpreting F ratios

F table
at num df = 1
~~and denom df = 8~~

F .25 1.54
F .10 3.46
F .05 5.32
F .025 7.57
F .01 11.26

- F ratios confirm that factors A and B are significant at the 1% level.
- F ratio shows there is not a significant interaction.

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Design Matrix

									Replicates		
	A	B	C	AB	AC	BC	ABC	I	II	Total	Average
1	-	-	-	+	+	+	-				
2	+	-	-	-	-	+	+				
3	-	+	-	-	+	-	+				
4	+	+	-	+	-	-	-				
5	-	-	+	+	-	-	+				
6	+	-	+	-	+	-	-				
7	-	+	+	-	-	+	-				
8	+	+	+	+	+	+	+				
Avg +											
Avg -											
Effect											

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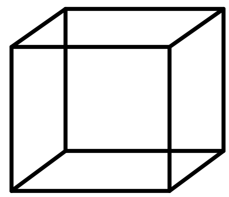
Cube Plot



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Why use 2^k designs ?

- Easy to use and data analysis can be performed using graphical methods.
- Relatively few runs required.
- 2^k designs have been found to meet the majority of the experimental needs of those involved in the improvement of quality.
- 2^k designs are easy to use in sequential experimentation.
- Fractions of the 2^k (fractional factorials) can be used to further reduce the experiment size.

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Moen, Nolan and Provost, Improving Quality Through Planned Experimentation