Example 11.1

Simulation with Built-In Excel Tools

Background Information

- In August, Walton Bookstore must decide how many of next year's nature calendars to order.
- Each calendar costs the bookstore $7.50 and is sold for $10.
- After February 1 all unsold calendars are returned to the publisher for a refund of $2.50 per calendar.

Background Information -- continued

- Walton believes that the number of calendars it can sell by February 1 follows this probability distribution.

Probability Distribution of Demand for Walton Example

Calendars Demanded	Probability
100	0.30
150	0.20
200	0.30
250	0.15
300	0.05

- Walton wants to maximize the expected profit from calendar sales.

Solution

- We first discuss the probability distribution in the table.
- It is a **discrete** distribution with only five possible values: 100, 150, 200, 250 and 300.
- In reality, it is clear that other values of demand are possible.
- In spite of its apparent lack of realism, we use this discrete distribution for two reasons.

Solution -- continued

- First, its simplicity is a nice feature to get us started with simulation modeling.
- Second, discrete distributions are often used in real business simulation models.
- Even though discrete distribution is only an approximation to reality, it can still give us important insights into the actual problem.
- As for the probabilities in the table, they are typically drawn from historical data or educated guesses.

WALTON1.XLS

- For a fixed order quantity, we will show how Excel can be used to simulate 1000 replications (or any other number of replications).
- Each replication is an independent replay of the events that occur.
- To illustrate, suppose we want to estimate the expected profit if Walton orders 200 calendars. To do this we need to simulate 1000 independent simulations.
- This file contains the setup needed to begin the simulation.

Developing The Simulation Model

- To develop the model, use the following steps**.**
  - **Inputs:** Enter the cost data in the range B4:B6, the probability distribution of demand in the range E5:F9, and the proposed order quantity, 200, in cell B9. Columns E and F contain the demand values and the individual probabilities. It is also convenient to have the cumulative probabilities in column D. To obtain these, first enter the value 0 in cell D5. Then enter the formula **=F5+D5** in cell D6 and copy it to the range D7:D9.
  - **Generate Random Number:** Enter a random number in cell B19 with the formula **=RAND( )** and copy it to the range B20:B1018. Then freeze the random numbers in this range.

Developing The Simulation Model -- continued

- **Generate demands:** The key to the simulation is the generation of the customers demands in the range C19:C1018 from the random numbers in column B and the probability distribution of demand. To do this we:
  - Divide the interval from 0 to 1 into five segments. The lengths of the segments relate to the probabilities of various demands.
  - Then we associate a demand with each random number depending on which interval the random number falls into.

Developing The Simulation Model -- continued

- To accomplish this we can follow one of two ways:
  - The first is to use a nested IF statement in cell C19 (and copy it down C).
  - The second and simpler way is to use the VLOOKUP function. To do this we create a "lookup table" in the range D5:E9 and name it Lookup. Then enter the formula
    =VLOOKUP(B19,Lookup,2)
    in cell C19 and copy it to the range C20:C1018. The function compares the random number to the values in D5:D9 and returns the appropriate demand in E5:E9.
- **Revenue:** Once the demand is known, the number of calendars sold is the smaller of the demand and the order quantity. To calculate revenue for the first replication in D13 we enter **=UnitPrice*MIN(C19,OrderQuan)**.

Developing The Simulation Model -- continued

- **Ordering Cost:** The cost of ordering the calendars does not depend on the demand; it is the unit cost multiplied by the number ordered. Calculate this in cell E19 with the formula **=UnitCost*OrderQuan**.
- **Refund:** If the order quantity is greater than the demand, there is a refund of $2.50 for each calendar left over, otherwise there is no refund. Therefore, enter the total refund for the first replication in cell F19 with the formula **=UnitRefund*MAX(OrderQuan-C19,0)**.
- **Profit:** Calculate the profit for this replication in G19 with the formula **=D19-E19+F19**.

Developing The Simulation Model -- continued

- **Copy to other rows:** Do the same bookkeeping for the other 999 replications by copying the range D19:G19 to the range D20:G1018.
- **Summary Measures:** Each profit value in column G corresponds to one randomly generated demand. First, calculate the average and standard deviation of the 1000 profits in cells B12 and B13 with the formulas **=AVERAGE(Profits)** and **=STDEV(Profits)**. Similarly, calculate the smallest and largest profit with the MIN and MAX functions.

Developing The Simulation Model -- continued

- **Confidence Interval for expected profit:** Finally, calculate a 95% confidence interval for the expected profit in cells E13 and E14 with the formulas
  =AvgProfit-1.96*StDevProfit/SQRT(1000)
  =AvgProfit+1.96*StDevProfit/SQRT(1000)

- At this point we need to look and see what we have accomplished.
- Let's look at the results of the simulation.

	A	B	C	D	E	F	G	H	I
1	**Simulation of Walton's bookstore**								
2									
3	**Cost data**			**Demand distribution**					
4	Unit cost	$7.50		Cum Prob	Demand	Probability			
5	Unit price	$10.00		0.00	100	0.30			
6	Unit refund	$2.50		0.30	150	0.20			
7				0.50	200	0.30			
8	**Decision variable**			0.80	250	0.15			
9	Order quantity	200		0.95	300	0.05			
10									
11	**Summary measures for simulation below**								
12	Average profit	$197.38		95% confidence interval for expected profit					
13	Stdev of profit	$321.82		Lower limit	$177.43				
14	Minimum profit	-$250.00		Upper limit	$217.32				
15	Maximum profit	$500.00							
16									
17	**Simulation**								
18	Replication	Random #	Demand	Revenue	Cost	Refund	Profit		
19	1	0.5279	200	$2,000	$1,500	$0	$500		
20	2	0.9117	250	$2,000	$1,500	$0	$500		
21	3	0.5765	200	$2,000	$1,500	$0	$500		
22	4	0.0251	100	$1,000	$1,500	$250	-$250		
23	5	0.1877	100	$1,000	$1,500	$250	-$250		
24	6	0.3948	150	$1,500	$1,500	$125	$125		
25	7	0.2403	100	$1,000	$1,500	$250	-$250		
26	8	0.1169	100	$1,000	$1,500	$250	-$250		
27	9	0.6386	200	$2,000	$1,500	$0	$500		
28	10	0.6780	200	$2,000	$1,500	$0	$500		
1012	994	0.8798	250	$2,000	$1,500	$0	$500		
1013	995	0.5215	200	$2,000	$1,500	$0	$500		
1014	996	0.3815	150	$1,500	$1,500	$125	$125		
1015	997	0.1507	100	$1,000	$1,500	$250	-$250		
1016	998	0.3880	150	$1,500	$1,500	$125	$125		
1017	999	0.3753	150	$1,500	$1,500	$125	$125		
1018	1000	0.4464	150	$1,500	$1,500	$125	$125		

Range names used:
UnitCost - B4
UnitPrice - B5
UnitRefund - B6
Lookup - D5:E9
OrderQuan - B9
AvgProfit - B12
StdevProfit - B13
Profits - G19:G1018

11.2 | 11.3 | 11.4 | 11.5

Accomplishments

- So here is what we have accomplished:
  - In the body of the simulation rows 19-1018, we randomly generated 1000 possible demands and the corresponding profits.
  - There are only five possible demand values and also for our order quantity, 200, the profit is $500 regardless of whether demand is 200, 250, or 300.
  - There are 290 trials with profit equal to - $250, 227 trials with profit equal to $125, and 483 trials with profit equal to $500.
  - The average of the 1000 profits is $197.38 and their standard deviation is $328.58. (Answers may differ because of the random numbers.)

11.2 | 11.3 | 11.4 | 11.5

Probability Distributions

- The probability distribution of profit is as follows:
  - *P*(Profit = -$250) = 290/1000 = 0.29
  - *P*(Profit = -$125) = 227/1000 = 0.227
  - *P*(Profit = -$500) = 483/1000 = 0.483
- We also estimate the mean of this distribution to be $197.83 and its standard deviation to be $321.82.
- It is important to be aware that with computer simulation each time it is run the answers will be slightly different.
- This is the reason for the confidence interval.

Confidence Intervals

- The confidence intervals can be found in cells E13 and E14.
- This interval expresses our uncertainty about the mean of the profit distribution.
- Our best guess is the value we observed but because the corresponding confidence interval is very wide, from $177.43 to $217.32, we are not sure of the true mean of the profit distribution.

Confidence Intervals -- continued

- It is common in computer simulation to estimate the mean of some distribution by the average of 1000 profits.
- The usual practice is then to accompany this estimate with a **confidence interval**, which indicates the accuracy of the estimate.
- You might recall from statistics that to obtain a confidence interval for the mean, you start with the estimated mean and then add and subtract a multiple of the **standard error** of the estimated mean.

Finding the Best Order Quantity

- So far we have ran the simulation for only a single order quantity, 200.
- Walton's ultimate goal is to find the best order quantity - that is, the order quantity that maximizes the mean profit.
- This goal can be achieved by using a data table to rerun the simulation for other order quantities. The data table can be found in the WALTON1.XLS file.

Using the Data Table

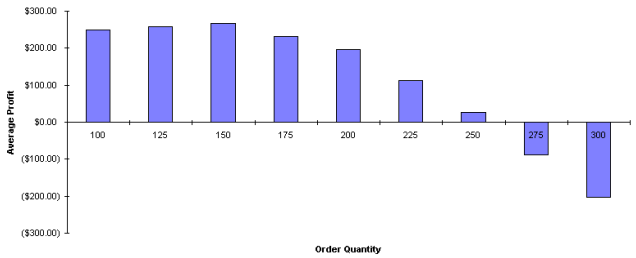
	A	B	C	D	E
1020	**Data table for average profit versus order quantity**				
1021	Order quantity	AvgProfit			
1022		$197.38			
1023	100	$250.00			
1024	125	$258.13			
1025	150	$266.25			
1026	175	$231.81			
1027	200	$197.38			
1028	225	$112.31			
1029	250	$27.25			
1030	275	($88.38)			
1031	300	($204.00)			

- To form this table, enter the trial order quantities in A1023:A1031, enter the formula **=AvgProfit** in cell B1022, and select the data table range.

11.2 | 11.3 | 11.4 | 11.5

Using the Data Table -- continued

- Use the Data/Table command, specifying that the single (column) input cell is B9.
- Construct a bar chart (shown below) of the average profits in the data table.



11.2 | 11.3 | 11.4 | 11.5

Results

- An order quantity of 150 appears to maximize profits in the data.
- However, keep in mind this is a simulation, so that all of these average profits depend on the particular random numbers generated.

To Freeze or Not To Freeze

- In developing this simulation, we suggested that you freeze the random numbers in column B.
- If you neglect this step, every time you press the F9 key or make any change to your spreadsheet model, a new set of simulated answers will appear.
- However, the drawback is that once the random numbers are frozen, you are stuck with that particular set of random numbers.

WALTON2.XLS

- This file is setup to illustrate another method that is more general.
- The other method uses a data table to generate the replications.
- Through row 19 this file and method are the same.
- The next step, however, is different. We form a data table in the range A23:B1023 to replicate the basic simulation 1000 times.

11.2 | 11.3 | 11.4 | 11.5

Data Table Method

- In column A we list the replication of numbers, 1-1000.
- The formula for the data table in cell B23 is **=Profit**. This copies the profit in the prototype row for use in the data table.
- Then we use the Data/Table command with *any blank cell* as the column input.
- Excel repeats the row 19 calculations 1000 times, each time with a new random number.
- Each time the profit is reported.

11.2 | 11.3 | 11.4 | 11.5

How the Data Table Works

- To understand this procedure we need to understand how the data table is formed.
- Excel takes each value in the left-hand column of the data table, substitutes it into the cell we designate, recalculates the spreadsheet, and returns the “bottom line” value we’ve requested in the top row of the data table.
- This process requires that we do not freeze the cell the random number is in.

WALTON3.XLS

- To take this one step further, we can use a two-way data table to see how the profit depends on the order quantity.
- The two-way data table has the replication number down the side and the possible order quantities along the top. This file contains the setup of the data table.
- The driving formula is in A23, is again **=Profit** and the column input is a blank cell, but this time the row input is B9.
- The following slide shows the average profit versus order quantity using a data table

	A	B	C	D	E	F	G	H	I
1	**Simulation of Walton's bookstore**								
2									
3	**Cost data**			**Demand distribution**					
4	Unit cost	$7.50		CumProb	Demand	Probability			
5	Unit price	$10.00		0.00	100	0.30			
6	Unit refund	$2.50		0.30	150	0.20			
7				0.50	200	0.30			
8	**Decision variable**			0.80	250	0.15			
9	Order quantity	200		0.95	300	0.05			
10									
11	**Summary measures from simulation below**								
12	Average	$198.13		95% confidence interval for expected profit					
13	StDev	$328.15		Lower limit	$177.79				
14	Minimum	-$250.00		Upper limit	$218.46				
15	Maximum	$500.00							
16									
17	**Simulation**								
18	Demand	Revenue	Cost	Refund	Profit				
19	250	$2,000	$1,500	$0	$500				
20									
21	**Data table for replications, each shows profit from that replication**								
22	Replication	Profit							
23		$500							
24	1	$125							
25	2	$500							
26	3	$125							
27	4	$500							
28	5	$500							
29	6	-$250							
30	7	$500							
31	8	$500							
32	9	-$250							
33	10	-$250							
1018	995	$500							
1019	996	-$250							
1020	997	-$250							
1021	998	-$250							
1022	999	$500							
1023	1000	$500							

Range names used:
UnitCost - B4
UnitPrice - B5
UnitRefund - B6
Lookup - D5:E9
OrderQuan - B9
AvgProfit - B12
StdevProfit - B13
Demand - A19
Profit - E19
Profits - B24:B1023

11.2 | 11.3 | 11.4 | 11.5

Two-Way Data Table Results

- After averaging the numbers in each column of the table, we see that 150 appears to be the best order quantity again.
- It is also helpful to construct a bar chart of these averages.
- To see if 150 is really the best, you can keep pressing F9 and the spreadsheet will recalculate and so will the output and the bar chart.

11.2 | 11.3 | 11.4 | 11.5