Example 13.1

Inventory Models

Background Information

- Machey's Department Store sells 1200 cameras per year, and the demand pattern throughout the year is very steady.
- The store orders its cameras from the regional warehouse, and it usually takes a week for the cameras to arrive after an order has been placed.
- Each time an order is placed, an ordering cost of $35 is incurred. The store pays $100 for each camera and sells them for $130 apiece.

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Background Information -- continued

- There is no physical storage cost, but the store's annual cost of capital is estimated at 10% per year – that is, it can earn 10% on any excess cash it invests.
- The store wants to determine how often it should order cameras, when it should place orders, and how many cameras it should order in each order.

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Solution

- We first discuss some basic quantities and relationships.
- Let D=1200 be the annual demand. Because it occurs steadily throughout the year, Machey's will place an order for Q cameras every time it is about to run out.
- Therefore, the only decision variable is Q, the order quantity. Once we know Q, the number of orders per years is given by D/Q.

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Solution -- continued

- Equivalently, the time between orders is *Q*/*D*.
- The key aspect is that the inventory jumps up to *Q* whenever an order arrives and decreases linearly until the next order arrives.

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Solution -- continued

- The problem is to find an order quantity *Q* that maximizes Machey's annual profit.
- There are several components of the annual profit.
- First, each time Machey's places an order, it incurs a fixed ordering cost, labeled *K*. For this example *K* = $35.
- Because there are *D/Q* placed per year, the annual ordering cost is *K D/Q* .

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Solution -- continued

- On top of this, Machey's pays a variable cost, labeled c, for each camera it purchases. Hence c = $100.
- Because the annual demand is D = 1200 and all demand must be met, the annual variable cost is cD = $120,000.
- Similarly, the company's revenue from each camera, labeled r, is r = $130, so its annual revenue is rD.
- Now we consider the annual holding cost.

Solution -- continued

- There is no cost for physically storing cash tied up in inventory. If we let i be Machey's annual cost of capital, where i = 0.10, then it can be shown from a net present value (NPV) argument that the relevant annual holding cost is i multiplied by the average monetary value of inventory, where this average is over the entire year.
- Because the inventory decreases linearly from Q to 0 between orders, the average level of the inventory at a typical point in time is (Q+0)/2 = Q/2, which implies that the average monetary value is $cQ/2$. Therefore, the annual holding cost from money tied up in inventory is $icQ/2$.

EOQ1.XLS

- The spreadsheet model appears on the next slide.
- This file contains the model.

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	A	B	C	D	E	F
1	**Machey's EOQ model**					
2						
3	**Inputs**					
4	Fixed ordering cost	$35				
5	Annual interest rate	10%		**Range names used:**		
6	Unit purchasing cost	$100		FixedCost - B4		
7	Selling price per unit	$130		IntRate - B5		
8	Annual demand	1200		UnitPurchCost - B6		
9	Lead time in years	1/52		UnitPrice - B7		
10				AnnDemand - B8		
11	**Ordering model**			LeadTime - B9		
12	Order quantity Q	91.65		OrderQuan - B12		
13	Orders per year	13.09		OrdersPerYear - B13		
14	Time between orders (days)	27.88		AnnProfit - B21		
15						
16	**Monetary values**					
17	Annual fixed ordering cost	$458		affected by order quantity		
18	Annual holding cost	$458				
19	Annual purchasing cost	$120,000		unaffected by order quantity		
20	Annual revenue	$156,000				
21	Annual profit	$35,083				
22						
23	**Alternative EOQ formula**	91.65				

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Developing the Simulation Model

- All of the formulas are based on the previous discussion.
- For example, the annual holding cost, determined by the equation for annual financial holding cost, is calculated in cell B18 with the formula **=InitRate*UnitPurchCost*OrderQuan/2**.
- Note that the only changing cell is the OrderQuan cell. It drives all of the quantities below it except for the annual purchase cost and the annual revenue, which do not depend on order quantity.

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Using the Solver

- We maximize annual profit with a single changing cell, the order quantity cell.
- There are no constraints other than nonnegativity of the order quantity.
- Also, the Assume Linear Model should not be checked. The reason is that the decision variable *Q* appears in the denominator of the annual ordering cost. This makes the model nonlinear.

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Solver Results

- The Solver solution specifies that Machey's should order 91 or 92 cameras each time it orders.
- This will result in about 13 orders per year, or about one order every 28 days.
- Note that the annual ordering cost and the annual financial holding cost for this optimal solution are equal. This is no coincidence.
- It can be shown with calculations that the Solver always chooses the order quantity that makes these two costs equal.

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EOQ Formula

- A feature of some nonlinear models, including the EOQ model is that they have no constraints and can be solved with calculus.
- Although we will not pursue the details, the calculus solution, shown in cell B23 of the model, is that the optimal order quantity satisfies

$$Q = \sqrt{2KD/(s+ic)}$$

- The advantage of this well-known "square-root formula" is that it gives us immediate insight into the effects of changes in inputs.

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EOQ Formula -- continued

- The disadvantage of this formula is that it holds only under the assumptions we have described.
- If a company wants to modify the EOQ model to meets any special circumstances, it would do better to develop a flexible spreadsheet model and then use the Solver instead of relying on a formula it does not fully understand.

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