Linear Programming

Graphical Technique

LINEAR PROGRAMMING

A problem solving approach in which we seek to minimize or maximize some objective function subject to a set of constraints that limit the degree to which the objective can be pursued.

Three main solution techniques:

- Graphical
- Simplex (algebraic)
- Computer

Vocabulary

- the maximization or minimization of a quantity is called the **objective**.
- there are **constraints** that limit the degree to which the objective can be pursued.
- the **feasible solution space** contains the solutions that satisfy all the constraints simultaneously.
- the **optimum solution** is the solution that meets the objective and will be an extreme point (corner point) of the feasible solution space.

Applications

- Integrated production, distribution and inventory planning at Libby-Owens-Ford resulting in $2 million savings per year.
- Media planning resulting in savings of 20-30%
- GE Capital -- used in managing delinquent accounts -- savings of $37 million per year.
- Optimizing flight crew scheduling at American Airlines -- savings of about $18 million per year.

Example: Graphical Solution

Softwater produces a pellet that can be sold in 40 and 80 pound bags. A common production line produces both products. Softwater expects the aggregate demand for next week to be 16,000 pounds. How many of the 40 # and 80 # bags should be produced ?

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Objective Function

Softtwater would like to maximize their profits.

Softwater makes $2 for every 40# bag it sells and $4 for every 80# bag.

Let X1 = 40# bag and X2 = 80# bag, then ---

Total profit = Z = 2 X1 + 4 X2

Objective function

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Constraints:

In a normal work week the production line operates 1500 minutes. Each 40# bag takes 1.2 minutes and each 80# bag takes 3 minutes.

Total packaging time = 1.2 X1 + 3 X2 <= 1500 minutes

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Constraints:

Softwater has 6000 square feet of packaging material available for next week. Each 40# bag takes 6 sq ft and each 80# bag takes 10 sq ft.

Total materials = 6 X1 + 10 X2 <= 6000 sq ft

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Constraints:

Aggregate demand is 16,000 pounds.

Demand = 40 X1 + 80 X2 => 16,000 pounds

Non-negativity constraint:

X1 and X2 must be at least 0.

Mathematical model:

Maximize Z = 2 X1 + 4 X2

subject to:

1.2 X1 + 3 X2 <= 1500

6 X1 + 10 X2 <= 6000

40 X1 + 80 X2 => 16000

X1, X2 => 0

Graphical Solution Procedure for Maximization problems:

1. Graph feasible solution points for each constraint.
2. Identify the feasible solution space.
3. Draw a line showing the x1 and x2 variables that yield a specified value of the objective function.
4. Move lines parallel to the line from #3 above through the solution space (away from the origin) until identifying the point in the feasible solution space that last touches the line -- this is the optimum point.
5. Solve algebraically for exact point.

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Identify end points for each constraint line and plot line.

Time

1.2 X1 + 3 X2 = 1500

X1 = 0, X2 = 500

X2 = 0, X1 = 1250

Demand

40 X1 + 80 X2 = 16000

x1 = 0, x2 = 200

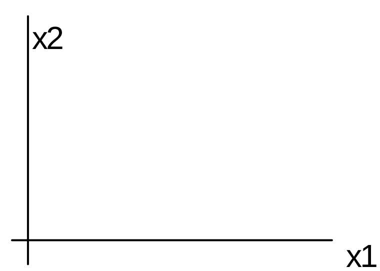
x2 = 0, x1 = 400

Materials

6 X1 + 10 X2 = 6000

X1 = 0, X2 = 600

X2 = 0, X1 = 1000



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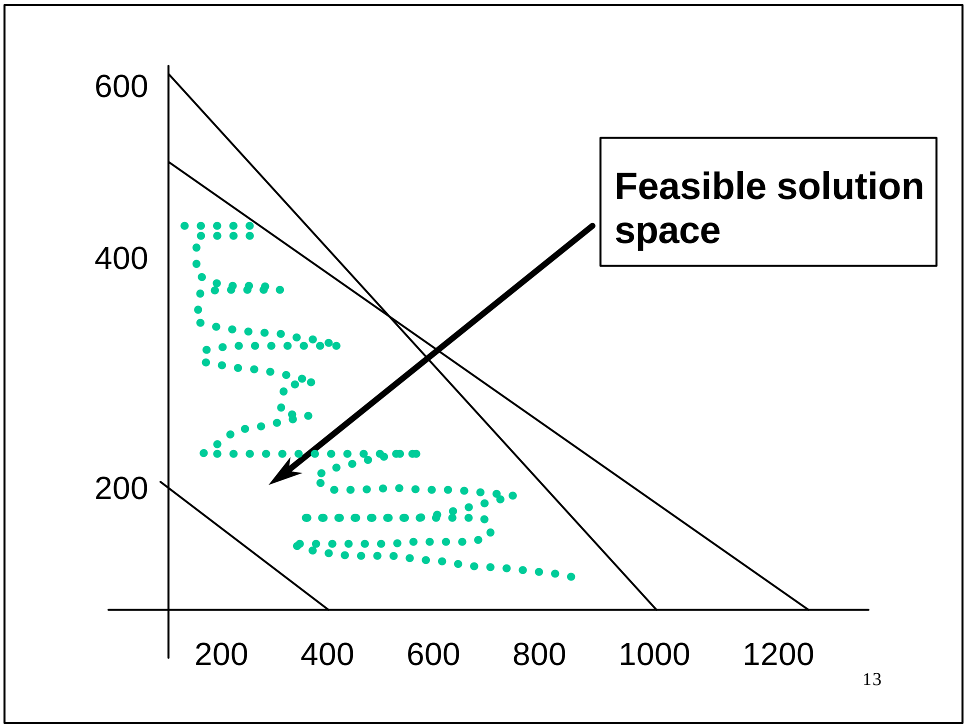
600
400
200
Feasible solution space
200
400
600
800
1000
1200
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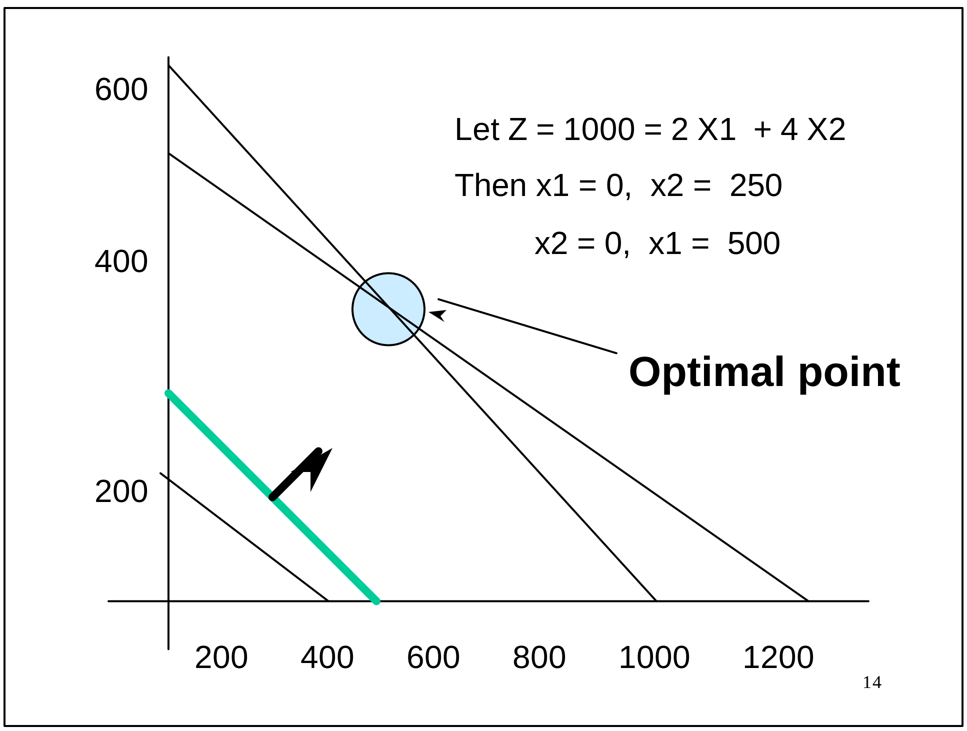
600
400
200
Let Z = 1000 = 2 X1 + 4 X2
Then x1 = 0, x2 = 250
x2 = 0, x1 = 500
Optimal point
200
400
600
800
1000
1200
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Formulate and solve the following LP problem:

A small construction firm specializes in building and selling single family homes. The firm offers two basic types of houses, model A and model B. Model A homes require 4,000 labor hours, 2 tons of stone, and 2,000 board feet of lumber. Model B houses require 10,000 labor hours, 3 tons of stone, and 2,000 board feet of lumber. Due to long lead times for ordering supplies and the scarcity of skilled and semi-skilled workers in the area, the firm will be forced to rely on its present resources for the upcoming building season. It has 400,000 hours of labor, 150 tons of stone, and 200,000 board feet of lumber. What mix of model A and B houses should the firm construct if model A's yield a profit of $1,000 per unit and model B's yield $2,000 per unit? Assume that the firm will be able to sell all the units it builds.

Solution: A = 37 (exact answer 37.5) and B = 25.

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