

Examples from Class 8/31/11

In these examples we computed the Fourier coefficients for some functions on $[-\pi, \pi]$, formed a (truncated) Fourier series, and plotted the results.

■ Example 1: $f(x) = e^x$

$$f[x_] = e^x;$$

$$a[0] = \frac{1}{\pi} \int_{-\pi}^{\pi} f[x] dx$$

$$\frac{-e^{-\pi} + e^{\pi}}{\pi}$$

$$a[n_] = \text{Simplify}\left[\frac{1}{\pi} \int_{-\pi}^{\pi} f[x] \text{Cos}[n x] dx, \text{Assumptions} \rightarrow n \in \text{Integers}\right]$$

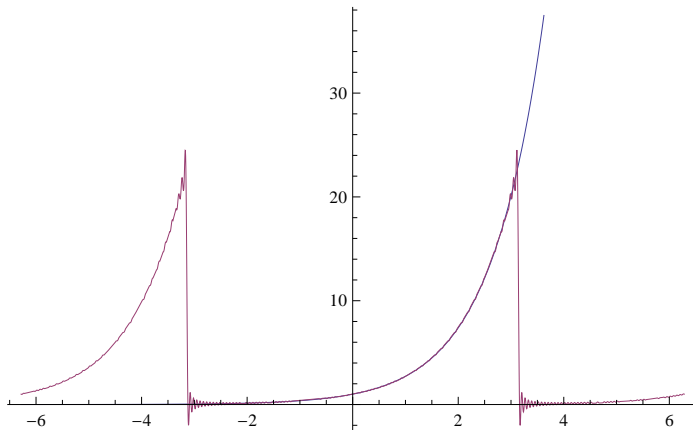
$$\frac{2 (-1)^n \text{Sinh}[\pi]}{\pi + n^2 \pi}$$

$$b[n_] = \text{Simplify}\left[\frac{1}{\pi} \int_{-\pi}^{\pi} f[x] \text{Sin}[n x] dx, \text{Assumptions} \rightarrow n \in \text{Integers}\right]$$

$$-\frac{2 (-1)^n n \text{Sinh}[\pi]}{\pi + n^2 \pi}$$

$$s[x_] = \frac{a[0]}{2} + \sum_{n=1}^{100} (a[n] \text{Cos}[n x] + b[n] \text{Sin}[n x]);$$

Plot[{ e^x , $s[x]$ }, { x , -2π , 2π }]



■ Example 2: $f(x) = |x|$

Clear[a, b, f, s]

$$f[x_] = \text{Abs}[x];$$

$$a[0] = \frac{1}{\pi} \int_{-\pi}^{\pi} f[x] dx$$

$$\pi$$

$$a[n_] = \text{Simplify}\left[\frac{1}{\pi} \int_{-\pi}^{\pi} f[x] \text{Cos}[n x] dx, \text{Assumptions} \rightarrow n \in \text{Integers}\right]$$

$$\frac{2(-1 + (-1)^n)}{n^2 \pi}$$

$$b[n_] = \text{Simplify}\left[\frac{1}{\pi} \int_{-\pi}^{\pi} f[x] \text{Sin}[n x] dx, \text{Assumptions} \rightarrow n \in \text{Integers}\right]$$

0

$$s[x_] = \frac{a[0]}{2} + \sum_{n=1}^{100} (a[n] \text{Cos}[n x] + b[n] \text{Sin}[n x]);$$

Plot[{Abs[x], S[x]}, {x, -2 π, 2 π}]

