Math/Phys 4530 Homework #1 Dr. S. Robinson

Fall 2011

Due Friday, September 9:

Some of these problems are sdapted from Riley, Hobson, and Bence, *Mathematical Methods for Physics and Engineering,* and Arfken and Weber, *Mathematical Methods for Physicists.*

1. Planck's theory of quantized oscillators leads to an average energy

$$\langle \varepsilon \rangle = \frac{\sum\limits_{n=1}^{\infty} n\varepsilon_0 e^{-n\varepsilon_0/(kT)}}{\sum\limits_{n=0}^{\infty} e^{-n\varepsilon_0/(kT)}}$$

where $\varepsilon_0 = h\nu$, h is the Planck constant, k is the Boltzmann constant, and T is the absolute temperature.

(a) Prove that the series in the denominator converges and

$$\sum_{n=0}^{\infty} e^{-n\varepsilon_0/(kT)} = \frac{1}{1 - e^{-\varepsilon_0/(kT)}}.$$

(b) Prove that the series in the numerator converges and

$$\sum_{n=1}^{\infty} n\varepsilon_0 e^{-n\varepsilon_0/(kT)} = \frac{e^{-\varepsilon_0/(kT)}\varepsilon_0}{(e^{-\varepsilon_0/(kT)} - 1)^2}.$$

(c) Conclude from parts (a) and (b) that

$$\langle \varepsilon \rangle = \frac{\varepsilon_0}{e^{\varepsilon_0/(kT)} - 1}.$$

(d) Show that, if $kT >> \varepsilon_0$ then $\langle \varepsilon \rangle \approx kT$, while if $kT << \varepsilon_0$ then $\langle \varepsilon \rangle \approx \varepsilon_0 e^{-\varepsilon_0/(kT)}$.

2. (a) Problem 27 on page 81 of the text.

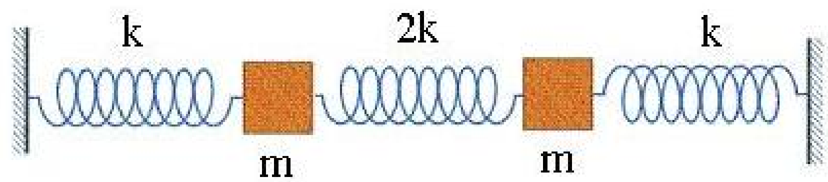
(b) Prove the identity

$$\left(\frac{ia - 1}{ia + 1}\right)^{ib} = e^{-2b \cot^{-1} a}$$

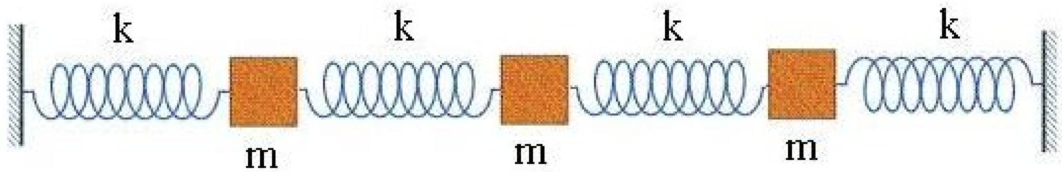
in which a and b are real. This identity arises in the quantum theory of photoionization.

3. Find the characteristic frequencies and characteristic modes of vibration for the following systems of masses and springs:

(a)



(b)



4. Problem 7 on page 355 of the text. Then expand the function in a series of complex exponentials e^{inx} on $(-\pi, \pi)$ and verify that the two series are equivalent.

5. Problem 23 on page 371 of the text.