Math/Phys 4530 Homework #4 Dr. S. Robinson

Fall 2011

Due Friday, November 18:

1. The mathematical model for the velocity potential $\phi(x, y)$ in the steady, two-dimensional, irrotational flow of an ideal fluid near a square corner is

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

for $0 < x < \pi$, $0 < y < \pi$, with the boundary conditions

$$\begin{aligned} \frac{\partial\phi}{\partial x}(0, y) &= 0 \\ \phi(\pi, y) &= c_1 \\ \frac{\partial\phi}{\partial y}(x, 0) &= 0 \\ \phi(x, \pi) &= c_2 \end{aligned}$$

where c_1 and c_2 are constants with $c_2 > c_1$. Find $\phi(x, y)$.

2. Solve the time-dependent Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi = -\frac{\hbar^2}{2m}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}\right)$$

for $\Psi = \Psi(r, \theta, t)$ where $0 < r < a$ and

$$\begin{aligned} \Psi(a, \theta, t) &= 0 \\ \lim_{r\to 0}|\Psi(r, \theta, t)| &< \infty \\ \Psi(r, \theta, 0) &= \frac{2}{a^2\sqrt{\pi}} r\sin\theta. \end{aligned}$$

3. Problem #5 on pages 662-663 of the text.

4. Problem #3 on page 658 of the text.

5. Show that the curve between the points (r_1, θ_1) and (r_2, θ_2) that minimizes

$$I = \int_{(r_1,\theta_1)}^{(r_2,\theta_2)} \frac{1}{r}\, ds$$

where

$$ds = \sqrt{dr^2 + r^2 d\theta^2}$$

is a logarithmic spiral.